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$N=2$ super W algebras and generalized $N=2$ super KdV hierarchies based on Lie superalgebras

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Abstract. We propose a hierarchy of super Lax equations based on an affine Lie superalgebra $A(n,n)^{(1)}$ as the generalized $N=2$ super KdV hierarchy and show that the associated super Gel'fand–Dikii bracket defines an $N=2$ super W algebra. For $A(2,2)^{(1)}$ we obtain the $N=2$ super Boussiensque equation and the $N=2$ super W_2 algebra (containing additional currents of spins 2, 5/2, 3).

1. Introduction

W algebras [1] and their supersymmetric extention (super W algebras for short) [2] provide viable means of relating integrable systems and conformal field theory models. $N=2$ super W algebras are particularly interesting from both physics and mathematics points of view. They provide a framework for dealing with superstring compactification. Recently a model of topological conformal field theory has been constructed by twisting the $N=2$ superconformal field theory [3]. This observation opens the possibility of constructing topological W algebras out of $N=2$ super W algebras.

In the bosonic case W_n algebras are related to the second Hamiltonian structure of KdV type equations [4–6]. For example, the Gel'fand–Dikii (GD) bracket defining the Hamiltonian structure of the KdV type hierarchy associated with $A_{n-1}^{(1)}$ gives the classical W_n algebra. Quantization of the second Hamiltonian structure can be achieved in terms of the free field representation, or the Miura transformation [7]. This connection of the Kac–Moody algebra (current algebra) and the W_n algebra is described in a transparent way by the Lie algebraic approach to KdV type equations by Drinfeld and Sokolov [5].

We have recently succeeded in supersymmetric extension of the Drinfeld–Sokolov method by relating generalized super KdV hierarchies to Lie superalgebras. We have derived the $N=1$ and $N=2$ super KdV equations based on the affine Lie superalgebras $C(2)^{(2)}$ and $A(1,1)^{(1)}$, respectively [8, 9]. In this paper we propose a hierarchy of super Lax equations based on $A(n,n)^{(1)}$ as the generalized $N=2$ super KdV hierarchy and show that the super GD structure of the hierarchy defines the claseical $N=2$ super W algebra. The construction is illustrated in the case of $A(2,2)^{(1)}$ by driving the $N=2$ super Boussienesque equation and the $N=2$ super W_2 algebra (containing an $N=2$ super multiplet of currents of spins $(2, 5/2, 5/2, 3)$ in addition to the $N=2$ super Virasoro generators).

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2. Generalized $N=2$ super KdV hierarchy of $A(n, n)$ type

The Lie algebraic approach provides a systematic method of constructing generalized KdV type equations [5]. One associates with the affine Lie algebra $A_{n-1}^{(1)}$ a scalar Lax operator of nth order

$$L_B = \partial^n + u_{n-2}(z)\partial^{n-2} + \ldots + u_1(z)\partial + u_0(z) \tag{2.1}$$

where $\partial = \partial/\partial z$. A hierarchy of time evolutions

$$\frac{\partial L_B}{\partial t_k} = [(L_B^{k/n})_+, L_B] \tag{2.2}$$

gives the generalized KdV hierarchy of A_{n-1} type. Here the subscript $+$ means the differential operator part of the pseudo-differential operator $L_B^{k/n}$. The hierarchy (2.2) can also be obtained as the n-reduction of KP hierarchy [10]. We recall that $u_i(z)$ are conserved currents of spin $n-i$ and generate W_n algebra in terms of the Gel'fand–Dikii bracket structure [11].

We will show that the Lie algebraic approach described above can be extended to supersymmetric cases by use of Lie superalgebras (LSA). A particularly interesting class of hierarchy of super Lax equations can be constructed based on the affine LSA $A(n-1, n-1)^{(1)}$, with which we will be concerned in this paper.

A simple Lie superalgebra $A(n-1, n-1)$ is defined by [12]

$$A(n-1, n-1) := \mathfrak{sl}(n|n)/\langle cI \rangle \tag{2.3}$$

where $\mathfrak{sl}(n|n)$ is the set of $(n|n)\times(n|n)$ supertraceless supermatrices. The Lie superalgebra $\mathfrak{sl}(n|n)$ has one-dimensional centre $\langle cI \rangle$, the unit matix I being supertraceless. We take a quotient of $\mathfrak{sl}(n|n)$ by this centre to define the simple Lie superalgebra $A(n-1, n-1)$. The corresponding affine Lie superalgebra $A(n-1, n-1)^{(1)}$ admits a purely fermionic simple root system [13], which is indispensable in constructing a manifestly spacetime supersymmetric integrable system [8, 9, 14, 15].

As will be described in our forthcoming paper [16], the scalar super Lax operator of $2n$th order

$$L_S = D^{2n} + U_{2n-2}(\hat{z})D^{2n-2} + \ldots + U_2(\hat{z})D^2 + U_1(\hat{z})D \tag{2.4}$$

is associated with $A(n-1, n-1)^{(1)}$. Here D is the superderivative in supercoordinates $\hat{z} = (z, \theta)$, and U_k is an even (odd) superfield with spin $2n - k/2$ for even (odd) k. A few remarks follow on the form of the super differential operator (2.4). The supertraceless condition implies the absence of D^{2n-1}-term. The lack of the constant term in (2.4) is due to the fact we have divided by the unit matrix to define $A(n-1, n-1)$ (see (2.3)). We consider a hierarchy of super Lax equations

$$\frac{\partial L_S}{\partial t_{2k}} = [(L_S^{2k/2n})_{>0}, L_S]. \tag{2.5}$$

One can show in the same way as the bosonic case [5] that these equations define consistent and commuting flows for 'even' time t_{2k}. (We will not consider the flows in odd time directions.) Here the subscript >0 means taking the strictly positive differential operator part of the super pseudo-differential operator $L_S^{2k/2n}$, (i.e. without the constant term in $L_S^{2k/2n}$) as the time evolution generator. This prescription is necessary for incorporating the spin 1 current U_{2n-2} and excluding the constant term in (2.4), and it differs from the super KP hierarchy of Manin and Radul [17], in which U_{2n-2} is set to zero.

The simplest case of the hierarchy (2.5) was discussed in [9], where we have seen that (2.5) based on $A(1,1)^{(1)}$ gives the $N=2$ supersymmetric extension of KdV hierarchy. We will show later that the integrable system (2.5) is $N=2$ supersymmetric in the sense that there are conserved super U_1 current and super energy momentum tensor whose Poisson brackets realize the $N=2$ super Virasoro algebra. Hence, the hierarchy (2.5) can be considered as the generalized $N=2$ super KdV hierarchy of $A(n-1, n-1)$ type.

The first non-trivial (i.e. containing higher spin currents) example of $N=2$ super KdV type equations arises from $A(2,2)^{(1)}$. It turns out to be the $N=2$ supersymmetric extension of the Boussinesque equation. To see this we consider the sixth-order super Lax operator

$$L = D^6 - UD^4 - VD^3 - RD^2 - SD. \tag{2.6}$$

The even ($N=1$) superfields U and R have integer spins 1 and 2, while the odd ones V and S have half-integer spins 3/2 and 5/2. The generalized $N=2$ super KdV hierarchy of $A(2,2)$-type is

$$\frac{\partial L}{\partial t_{2k}} = [(L^{2k/6})_{>0}, L] \qquad (k \neq 3l). \tag{2.7}$$

The first non-trivial time evolution is obtained by taking $k=2$. In this case

$$(L^{4/6})_{>0} = D^4 - \tfrac{2}{3}UD^2 - \tfrac{2}{3}VD. \tag{2.8}$$

Evaluating the commutator, we have

$$\frac{\partial U}{\partial t_4} = -D^4U + UD^2U + 2D^2R \tag{2.9}$$

$$\frac{\partial V}{\partial t_4} = -D^4V + UD^2V + VD^2U + 2D^2S \tag{2.10}$$

$$\frac{\partial R}{\partial t_4} = D^4R + \tfrac{2}{3}(-D^6U + UD^4U + VD^3U - UD^2R + RD^2U + SDU - VD^2V - VDR + 2VS) \tag{2.11}$$

$$\frac{\partial S}{\partial t_4} = D^4S + \tfrac{2}{3}(-D^6V - UD^2S + UD^4V + VD^3V + RD^2V - VDS + SDV). \tag{2.12}$$

Setting $U=R=0$ and introducing bosonic components $u_2(z)$ and $u_3(z)$ by $V=\theta u_2$ and $S=\theta u_3$, we recover the Boussinesque equation

$$\begin{aligned} \frac{\partial u_2}{\partial t_4} &= -\partial^2 u_2 + 2\partial u_3 \\ \frac{\partial u_3}{\partial t_4} &= \partial^2 u_3 - \tfrac{2}{3}\partial^3 u_2 - \tfrac{2}{3}u_2\partial_2. \end{aligned} \tag{2.13}$$

3. $N=2$ super W algebra from super Gel'fand–Dikii bracket

A Hamiltonian structure for the generalized $N=2$ super KdV hierarchy (2.5) can be introduced by a supersymmetric extension of the second Gel'fand–Dikii bracket (super

GD bracket). The super GD bracket defines a Poisson bracket on the space of functionals of superfields $U_k(\hat{z})$, $(k=1,\ldots,2n-2)$ appearing in (2.4). The fundamental brackets $\{U_k, U_l\}$ can be calculated by taking delta-function like functionals. We will show that the algebra generated by the superfields U_k with the super GD bracket structure gives a classical $N=2$ super W-algebra.

The definition of the super GD bracket is based on a duality of the algebra of super differential operators L and the super Volterra algebra of super pseudo-differential operatos of negative degree X defined by

$$\langle L, X\rangle := \int \mathrm{d}z\, \mathrm{d}\theta\, \mathrm{Res}(LX) \tag{3.1}$$

where Res stands for the coefficient of D^{-1}. We can define an infinitesimal (coadjoint) action of X on superdifferential operators by

$$V_X(L) := L(XL)_+ - (LX)_+ L. \tag{3.2}$$

Let us take a superdifferential operator

$$\bar{L} = D^{2n-1} + U_{2n-2}(\hat{z})D^{2n-3} + \ldots + U_2(\hat{z})D + U_1(\hat{z}) \tag{3.3}$$

obtained by factoring out a single superderivative D from the super Lax operator (2.4), $L_S = \bar{L}D$. It is helpful to think of the superfields $U_1(\hat{z}), \ldots, U_{2n-2}(\hat{z})$ as a coordinate system of the infinite dimensional space $\mathcal{M}_{2n}$ of superdifferential operators of the form (2.4). Notice that the generalized $N=2$ super KdV hierarchy (2.5) of type $A(n-1, n-1)$ defines commuting flows on $\mathcal{M}_{2n}$. Then, for any functional $F[U_i]$ on $\mathcal{M}_{2n}$, we assign an element of super Volterra algebra

$$X_F := \sum_{k=1}^{2n-1} D^{-k} \cdot X_k \qquad X_k := (-1)^k \frac{\delta F[U_i]}{\delta U_k} \qquad (1 \leqslant k \leqslant 2n-2). \tag{3.4}$$

X_{2n-1} is to be determined by the condition that the action V_{X_F} preserves the form of (3.3). Mathematically speaking, we define a map from the functional space on $\mathcal{M}_{2n}$ to the super Volterra algebra which is the dual of $\mathcal{M}_{2n}$. We are now in a position to define the super GD bracket which is a Poisson bracket on $\mathcal{M}_{2n}$. For functionals $F[U_i]$ and $G[U_i]$ we define their bracket by

$$\{F[U_i], G[U_i]\} := \langle V_{X_F}(\bar{L}), X_G\rangle. \tag{3.5}$$

The super Gel'fand–Dikii bracket (3.5) gives a Hamiltonian structure of the hierarchy (2.5), that is, the commuting flows on $\mathcal{M}_{2n}$ defined by (2.5) are the Hamiltonian flows with respect to the bracket (3.5). The algebra generated by U_k's is nonlinearly (at most quadratically) closed under the super GD bracket. The (super) Jacobi identities are automatic by the definition (3.5).

To make contact with the $N=2$ super Virasolo algebra, let us examine the brackets for the spin 1 superfield $U := U_{2n-2}$ and the spin-$\frac{3}{2}$ superfield $V := U_{2n-3}$. This calculation will also illustrate how to evaluate a bracket $\{U_k, U_l\}$ in general. Making a slight change of notation, we consider

$$\bar{L} = D^{2n-1} - UD^{2n-3} - VD^{2n-4} - RD^{2n-5} - SD^{2n-6} + \ldots \tag{3.6}$$

and

$$X_F = D^{-2n+3} \cdot X + D^{-2n+2} \cdot Y + D^{-2n+1} \cdot Z \tag{3.7}$$

where we have omitted the irrelevant terms. A straightforward calculation of pseudo-differential operators gives

$$V_{X_F}(\bar{L}) = [n(n-1)Y^{(3)} + \tfrac{1}{2}n(n-1)X^{(4)} + (UX)^{(2)} + VX^{(1)} + Y(2V + U^{(1)})]D^{2n-3}$$
$$+ [\tfrac{1}{2}n(n-1)Y^{(4)} - UY^{(2)} + 2VX^{(2)} + XV^{(2)} - (YV)^{(1)}]D^{2n-4} + \dots \tag{3.8}$$

where we have used

$$Z^{(1)} = (XU)^{(1)} - (n-1)X^{(3)} - (n-1)Y^{(2)} \tag{3.9}$$

which is the condition that the D^{2n-2} term in $V_{X_F}(\bar{L})$ vanishes. We have used the notation $A^{(n)} := D^n A$ for superderivatives of superfield A. By (3.5) and (3.8) we can calculate the brackets by taking delta-function-like functionals for F and G:

$$\begin{aligned}
\{U(\hat{z}_1), U(\hat{z}_2)\} &= (n(n-1)D^3 - 2V(\hat{z}_2) + U^{(1)}(\hat{z}_2))\delta(\hat{z}_1 - \hat{z}_2) \\
\{V(\hat{z}_1), U(\hat{z}_2)\} &= (\tfrac{1}{2}n(n-1)D^4 - U(\hat{z}_2)D^2 + V(\hat{z}_2)D + U^{(2)}(\hat{z}_2))\delta(\hat{z}_1 - \hat{z}_2) \\
\{U(\hat{z}_1), V(\hat{z}_2)\} &= (\tfrac{1}{2}n(n-1)D^4 + U(\hat{z}_2)D^2 - V(\hat{z}_2)D - V^{(1)}(\hat{z}_2))\delta(\hat{z}_1 - \hat{z}_2) \\
\{V(\hat{z}_1), V(\hat{z}_2)\} &= (2V(\hat{z}_2)D^2 + V^{(2)}(\hat{z}_2))\delta(\hat{z}_1 - \hat{z}_2)
\end{aligned} \tag{3.10}$$

where $\delta(\hat{z}_1 - \hat{z}_2) = \delta(z_1 - z_2)(\theta_1 - \theta_2)$. D is the derivative with respect to $\hat{z}_1$. If we set

$$T = V - \tfrac{1}{2}DU \tag{3.11}$$

(3.10) takes the form

$$\{U(\hat{z}_1), U(\hat{z}_2)\} = (n(n-1)D^3 - 2T(\hat{z}_2))\delta(\hat{z}_1 - \hat{z}_2) \tag{3.12}$$

$$\{T(\hat{z}_1), U(\hat{z}_2)\} = (-U(\hat{z}_2)D^2 + \tfrac{1}{2}U^{(1)}(\hat{z}_2)D + U^{(2)}(\hat{z}_2))\delta(\hat{z}_1 - \hat{z}_2) \tag{3.13}$$

$$\{T(\hat{z}_1), T(\hat{z}_2)\} = (-\tfrac{1}{4}n(n-1)D^5 + \tfrac{3}{2}T(\hat{z}_2)D^2 + \tfrac{1}{2}T^{(1)}(\hat{z}_2)D - T^{(2)}(\hat{z}_2))\delta(\hat{z}_1 - \hat{z}_2). \tag{3.14}$$

It is easy to see that the algebra (3.12)-(3.14) in terms of Poisson brackets defines the classicla analogue of the $N = 2$ super Virasoro algebra. To this end we identify $\{U(\hat{z}_1), U(\hat{z}_2)\}$, etc with operator product expansions $U(\hat{z}_1)U(\hat{z}_2)$, etc and make the following identification:

$$\begin{aligned}
\delta(\hat{z}_1 - \hat{z}_2) &\leftrightarrow \theta_{12}(z_{12})^{-1} \\
D\delta(\hat{z}_1 - \hat{z}_2) &\leftrightarrow (z_{12})^{-1} \\
D^2\delta(\hat{z}_1 - \hat{z}_2) &\leftrightarrow -\theta_{12}(z_{12})^{-2} \\
&\vdots \\
D^5\delta(\hat{z}_1 - \hat{z}_2) &\leftrightarrow 2(z_{12})^{-3}.
\end{aligned} \tag{3.15}$$

We then obtain the operator product expansion of the $N = 2$ superconformal field theory. The resulting theory has the central charge $c = -3n(n-1)$.

The algebra we have obtained is an extension of the $N = 2$ super Virasoro algebra incorporating generators of spin $1, 3/2, \dots, n - 1/2$. This algebra, nonlinearly closed under the super GD bracket, is a classical $N = 2$ super W algebra. The generators other than U and T should form $N = 2$ super multiplets after taking appropriate combinations of them. To work out this explicity is quite involved for an arbitrary n. Here we present an example of the full structure of $N = 2$ super W algebra we have constructed in the case of $A(2, 2)^{(1)}$. We take

$$\bar{L} = D^5 - UD^3 - VD^2 - RD - S \tag{3.16}$$

and

$$X_F = D^{-1}\cdot X_1 + D^{-2}\cdot X_2 + D^{-3}\cdot X_3 + D^{-4}\cdot X_4 + D^{-5}\cdot X_5 \qquad (3.17)$$

where, for any functional $F[U, V, R, S]$, we put

$$X_1 = -\frac{\delta F}{\delta S} \qquad X_2 = \frac{\delta F}{\delta R} \qquad X_3 = -\frac{\delta F}{\delta V} \qquad X_4 = \frac{\delta F}{\delta U}. \qquad (3.18)$$

X_5 is to be determined by the condition that the terms of D^4 in V_{X_F} vanishes:

$$X_5 = -X_1^{(4)} - X_2^{(3)} + 2X_3^{(2)} + 2X_4^{(1)} - (X_1U)^{(2)} - (X_2U)^{(1)} + X_3U + (VX_1)^{(1)} + X_1R. \qquad (3.19)$$

Substituting this relation, $V_{X_F}(\bar{L})$ expressed in terms of $X_1 \ldots X_4$, U, V, R and S. The explicit form of $V_{X_F}(\bar{L})$ is very lengthy and is not given here.

To find the primary combination with respect to $N=2$ superconformal symmetry, we look at the brackets between the higher spin superfields R, S and $N=2$ super energy-momentum tensor U, T. These brackets are

$$\{U(\hat{z}_1), R(\hat{z}_2)\} = [-3D^5 - 3U(\hat{z}_2)D^3 + V(\hat{z}_2)D^2 + R^{(1)}(\hat{z}_2) - 2S(\hat{z}_2)]\delta(\hat{z}_1 - \hat{z}_2) \qquad (3.20)$$

$$\begin{aligned}\{T(\hat{z}_1), R(\hat{z}_2)\} \\ &= [\tfrac{1}{2}D^6 + \tfrac{1}{2}U(\hat{z}_2)D^4 - \tfrac{1}{2}V(\hat{z}_2)D^3 - 2R(\hat{z}_2)D^2 \\ &\quad + \tfrac{1}{2}R^{(1)}(\hat{z}_2)D + R^{(2)}(\hat{z}_2)]\delta(\hat{z}_1 - \hat{z}_2)\end{aligned} \qquad (3.21)$$

$$\{U(\hat{z}_1), S(\hat{z}_2)\} = [-2D^6 - 2U(\hat{z}_2)D^4 - 2V(\hat{z}_2)D^3 + 2R(\hat{z}_2)D^2 - S(\hat{z}_2)D]\delta(\hat{z}_1 - \hat{z}_2) \qquad (3.22)$$

$$\begin{aligned}\{T(\hat{z}_1), S(\hat{z}_2)\} \\ &= [D^7 + U(\hat{z}_2)D^5 - V(\hat{z}_2)D^4 - R(\hat{z}_2)D^3 + \tfrac{5}{2}S(\hat{z}_2)D^2 \\ &\quad + \tfrac{1}{2}S^{(1)}(\hat{z}_2)D - S^{(2)}(\hat{z}_2)]\delta(\hat{z}_1 - \hat{z}_2).\end{aligned} \qquad (3.23)$$

We find that the combinations

$$\begin{aligned} W_2 &= R - \tfrac{1}{3}T^{(1)} - \tfrac{1}{2}U^{(2)} + \tfrac{2}{9}U^2 \\ W_{5/2} &= S - \tfrac{1}{2}R^{(1)} - \tfrac{1}{2}T^{(2)} - \tfrac{1}{12}U^{(3)} + \tfrac{4}{9}UT \end{aligned} \qquad (3.24)$$

are the primary fields whose brackets with U and T take the desired form:

$$\{U(\hat{z}_i), W_2(\hat{z}_2)\} = -2W_{5/2}(\hat{z}_2)\delta(\hat{z}_1 - \hat{z}_2) \qquad (3.25)$$

$$\{T(\hat{z}_1), W_2(\hat{z}_2)\} = [-2W_2(\hat{z}_2)D^2 + \tfrac{1}{2}W_2^{(1)}(\hat{z}_2)D + W_2^{(2)}(\hat{z}_2)]\delta(\hat{z}_1 - \hat{z}_2) \qquad (3.26)$$

$$\{U(\hat{z}_1), W_{5/2}(\hat{z}_2)\} = [2W_2(\hat{z}_2)D^2 - \tfrac{1}{2}W_2^{(1)}(\hat{z}_2)D - \tfrac{1}{2}W_2^{(2)}(\hat{z}_2)]\delta(\hat{z}_1 - \hat{z}_2) \qquad (3.27)$$

$$\{T(\hat{z}_1), W_{5/2}(\hat{z}_2)\} = [\tfrac{5}{2}W_{5/2}(\hat{z}_2)D^2 + \tfrac{1}{2}W_{5/2}^{(1)}(\hat{z}_2)D - W_{5/2}^{(2)}(\hat{z}_2)]\delta(\hat{z}_1 - \hat{z}_2). \qquad (3.28)$$

The brackets of these primary fields are nonlinearly closed, for example

$$\begin{aligned}\{W_2(\hat{z}_1), W_2(\hat{z}_2)\} \\ &= [-\tfrac{2}{3}D^7 + \tfrac{4}{3}T(\hat{z}_2)D^4 + (\tfrac{16}{3}W_2 + \tfrac{8}{9}T^{(1)} + \tfrac{16}{27}U^2)(\hat{z}_2)D^3 \\ &\quad + (-\tfrac{5}{3}(W_2)^{(1)} - \tfrac{16}{9}T^{(2)} - \tfrac{8}{27}UU^{(1)})(\hat{z}_2)D^2 \\ &\quad + (-\tfrac{5}{3}W_2^{(2)} - \tfrac{4}{9}T^{(3)} - \tfrac{2}{27}UU^{(2)})(\hat{z}_2)D \\ &\quad + (W_2^{(3)} - 2W_2T + \tfrac{2}{3}T^{(4)} - \tfrac{2}{3}T^{(1)}T - \tfrac{4}{81}U^2T \\ &\quad + \tfrac{1}{3}U^{(2)}U^{(1)} + \tfrac{1}{27}U^{(3)}U + \tfrac{2}{9}UW_{5/2})(\hat{z}_2)]\delta(\hat{z}_1 - \hat{z}_2).\end{aligned} \qquad (3.29)$$

Since we have checked that $(W_2, W_{5/2})$ is an $N=2$ supermultiplet, the remaining brackets can be calculated by using super Jacobi identities.

4. Discussion

We have constructed the super Gel-fand–Dikii bracket associated with the generalized $N=2$ super KdV hierarchy of $A(n, n)$ type and have shown that it defines a classical $N=2$ super W algebra. Quantization of the super GD bracket and hence the quantum super W algebra can be achieved by use of the super Miura transformation, following the method developed by Fateev and Lukyanov [7] for bosonic W algebra. The connection of the resulting quantum $N=2$ super W algebra to the $N=2$ super W_∞ algebra [18] and to a topological W algebra [19] is an interesting problem.

In our Lie superalgebraic approach we have $N=1$ superfields. The appearance of $N=2$ supersymmetry is somewhat a mystery. In the approaches based on non-affine Lie superalgebras (Toda, WZNW model, soldering), one takes $A(n, n-1)$ to derive $N=2$ superconformal models [15, 20, 21]. The connection of this type of model to $N=2$ super coset models [22] has been studied recently [20]. Since the $A(n, n)$ and $A(N, n-1)$ have the same Cartan subalgebra, both can be used to obtain $N=2$ super W algebras. The $A(n, n)^{(1)}$ has a purely fermionic simple root system whereas the $A(n, n-1)^{(1)}$ does not, hence we have to switch to $A(n, n)^{(1)}$ to construct generalized $N=2$ super KdV hierarchies.

The $N=2$ super W algebra we have constructed consists of $N=2$ super multiplets of generators with zero $U(1)$ charge. It is an open question whether $N=2$ super W algebra containing $N=2$ super multiplets with non-zero $U(1)$ charge can be constructed based on the Lie superalgebraic method.

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Note added. After completing the manuscript of the present work we received preprints by Figueroa-O'Farrill and Ramos [23], in which $N=1$ and $N=2$ super W algebras are constructed with the same method in essence as used in our present and previous papers [9]. The connection of this method to affine Lie superalgebras is given in the present work.

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